

# Dynamical quantum non-locality

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During the 50 years since its discovery, the Aharonov–Bohm effect has had a significant impact on the development of physics. Its arguably deepest implication, however, has been virtually ignored.

Last year marked the fiftieth anniversary of the discovery of the Aharonov–Bohm (AB) effect<sup>1</sup>, one of the most surprising and quintessential effects in quantum mechanics. Since its discovery in 1959, the AB effect has made a significant impact on the development of physics. It was first proved experimentally by Robert Chambers<sup>2</sup>, merely one year after it was discovered, and since then the number of experiments and the variety of systems in which it has been observed has grown almost exponentially; from the spectacular electron microscopy experiments of Akira Tonomura, to magnetoresistance oscillations in mesoscopic physics, to quantum rings and many others — the list is steadily growing. On the theory side, it led to a deeper understanding of the nature of gauge theories in general as well as a better understanding of flux quantization in superconductors and of the quantum Hall effect. It has been generalized in a variety of directions — from a rather straightforward dual effect such as the Aharonov–Casher effect, to the celebrated Berry phase, to non-Abelian gauge theories, to Wilson loops, to anyons. During these past 50 years the impact of the AB effect has been significant indeed.

And during all these past 50 years, what I believe to be by far the deepest implication of the AB effect (discovered by Yakir Aharonov and described in his Tel Aviv University lecture notes and elsewhere<sup>3–5</sup>) has been virtually ignored. It is an implication that transcends the specific context from which it originates, and goes directly to the very core of quantum physics: the quantum equations of motion are non-local. Without appreciating this fact, it is safe to say that no real understanding of the nature of quantum mechanics is possible.

## Two kinds of non-locality

That nature is non-local, in the sense of so-called Bell-inequality violations, is by now well known. That is, quantum physics allows correlations between distant particles that are stronger than



Yakir Aharonov in December 2009, during his lecture at the ABB 50/25 conference in Bristol. He presented a paradox that highlighted a deep consequence of the Aharonov–Bohm effect: quantum dynamics is inherently non-local.

allowed classically: for classical systems to show such correlations they would need to communicate faster than light. Understanding this kind of non-locality and using it is one of the greatest achievements of quantum information science. The non-locality discovered in the AB effect, however, is completely different. Bell-inequality violations follow from the Hilbert-space structure of quantum mechanics; they are purely kinematic. On the other hand, the AB type of non-locality is dynamic — it is the non-locality of the quantum equations of motion.

The above observation begs one immediate comment: the Schrödinger equation, the basic equation of motion of quantum mechanics, is purely local. It contains only quantities that are local in space ( $\mathbf{r}$ ) and time ( $t$ ), namely the potential,  $V(\mathbf{r})$ , the wavefunction  $\Psi(\mathbf{r}, t)$ , and a finite number of its derivatives — a typical local equation. So what could we mean by saying

that the quantum equations of motion are non-local? The point is that looking at Schrödinger's equation and concluding from it that physics is local is an optical illusion: the Schrödinger equation refers to the wavefunction,  $\Psi$ , which is not directly observable. It is not even gauge invariant. The key is to look at the evolution of the observable physical variables, such as position and momentum — the so-called quantum observables — and at their equations of motion (technically the Heisenberg equations).

To understand the whole issue there is no better place to start than the most typical quantum set-up, the famous two-slit experiment. Consider a source that emits electrons (such as a hot wire out of which electrons evaporate). At some distance from the source there is a screen with two narrow slits and behind it there is a second screen covered with electron detectors. Electrons leave the source one by one and they end up at different points on the second screen. Imagine three experiments being carried out: the first with only one of the slits open, the second with the other slit open and the third in which both slits are open. Let  $N_1(x)$ ,  $N_2(x)$  and  $N_{12}(x)$  be the number of electrons that reach a position  $x$  on the screen in a certain amount of time in the three experiments. Unlike what we expect from classical mechanics,  $N_{12}(x) \neq N_1(x) + N_2(x)$ ; instead,  $N_{12}(x)$  exhibits a typical interference pattern, with minima and maxima. In particular, there are points at which fewer electrons arrive when both slits are open than when only a single slit was open. So somehow each electron knows that both slits are open. But how does an electron passing through one slit know if the other slit is open or not?

Quantum mechanics explains this experiment by a dual process. First, it associates a wavefunction with the electron. The wavefunction propagates from the source to the screen, passes through the two slits and exhibits an interference pattern at the screen. The mystery is what happens next: although the wavefunction covers the whole screen, the electron suddenly reincarnates, whole and

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undivided, at some arbitrary location. Where exactly the electron appears is random, but the probability for arriving at a point  $x$  is given by the absolute value squared of the wavefunction at that location. Whereas this dual process gives an accurate mathematical recipe for calculating the probabilities, it remains unexplained how exactly this 'electron reincarnation' happens, spurring an intense debate ever since the early days of quantum mechanics. It is one of the great mysteries of physics.

At the same time, the propagation of the wavefunction from the source to the screen received only minimal attention. It is viewed as a simple, ordinary wave interference phenomenon, not very different from interference of waves on water or of sound. It is anything but.

And it is here where the AB effect and ideas that Aharonov developed in later work<sup>3-5</sup> come in. Quantum wavefunction interference and classical wave interference differ in two essential ways. First, quantum mechanically the wavelength is associated with momentum. A change of the interference pattern leads to a change of momentum. (Classically a wave also carries momentum, but the spatial aspect of the wave doesn't have any direct relation to its momentum.) Second, the phase that governs the interference is local classically, but non-local quantum mechanically.

To be more specific, the wavefunction of the electron,  $\Psi(\mathbf{r})$ , can be written as a superposition of two terms, each describing a wave propagating through one of the slits:

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2}} \left( e^{i\alpha_1} \Psi_1(\mathbf{r}) + e^{i\alpha_2} \Psi_2(\mathbf{r}) \right) \quad (1)$$

The interference pattern that results at the screen depends on the relative phase  $\alpha = \alpha_2 - \alpha_1$ . Let us first consider waves on water, with the two-slit screen being a dam with two openings in front of the beach. Two observers on the dam, one next to each opening, could both observe and record the phase of the wave, by simply measuring how high the water is as a function of time. They could then find the relative phase by calculating the difference of the phases. So not only can the relative phase be observed, but the individual phases as well.

On the other hand, quantum mechanically  $\alpha_1$  and  $\alpha_2$  cannot be observed. Indeed, quantum mechanically an overall phase is unobservable. That is, the wavefunction given in equation (1) is indistinguishable from a wavefunction  $\Psi'(\mathbf{r})$  that differs from it by a global phase  $\beta$

$$\Psi'(\mathbf{r}) = e^{i\beta} \Psi(\mathbf{r}) = \frac{1}{\sqrt{2}} \left( e^{i(\alpha_1 + \beta)} \Psi_1(\mathbf{r}) + e^{i(\alpha_2 + \beta)} \Psi_2(\mathbf{r}) \right)$$

Hence the individual phases of the two wavepackets cannot be measured locally — they simply have no observational meaning. The relative phase is all there is. The consequences of this fact are far reaching. To see them, let us turn to the observables.

### A paradigmatic shift

Instead of working with the double-slit set-up as described above, for simplicity we will consider a one-dimensional situation: two non-overlapping wavepackets  $\Psi_1(x)$  and  $\Psi_2(x)$ . They are similar to the wavepackets describing the electron immediately after passing through the two slits in the double-slit experiment. In time, these two wavepackets will spread and interfere. Let the wavefunction at  $t = 0$  be

$$\Psi_\alpha(x) = \frac{1}{\sqrt{2}} \left( \Psi_1(x) + e^{i\alpha} \Psi_2(x) \right)$$

where the index  $\alpha$  reminds us what the relative phase is.

The interference pattern that will be produced when the two wavepackets overlap will obviously depend on the relative phase. But, at  $t = 0$ , what exactly are the observables that encode information about this phase?

Consider first the position  $x$ . The average of the position,  $\bar{x}$ , is given by

$$\begin{aligned} \bar{x} &= \int \Psi_\alpha^*(x) x \Psi_\alpha(x) dx = \\ &= \frac{1}{2} \int \Psi_1^*(x) x \Psi_1(x) dx + \\ &= \frac{1}{2} \int \Psi_2^*(x) x \Psi_2(x) dx + \\ &= \frac{1}{2} \int \Psi_1^*(x) x e^{i\alpha} \Psi_2(x) dx + \\ &= \frac{1}{2} \int e^{-i\alpha} \Psi_2^*(x) x \Psi_1(x) dx \end{aligned}$$

Clearly, only the last two integrals depend on  $\alpha$ . However, we chose  $\Psi_1$  and  $\Psi_2$  to be non-overlapping. Therefore, at any place  $x$  where  $\Psi_1$  is non-zero,  $\Psi_2$  is zero and vice versa. The integrands in the last two terms are therefore zero and the integrals vanish. As a consequence,  $\bar{x}$  is actually independent of  $\alpha$ . In a similar way we can see that for any power  $n$ , the average of  $x^n$  is independent of  $\alpha$ .

What about the momentum? The average of the momentum,  $\bar{p}$ , is given by the following equation:

$$\bar{p} = \int \Psi_\alpha^*(x) (-i\hbar) \frac{\partial}{\partial x} \Psi_\alpha(x) dx$$

Similar to above, the only dependence on  $\alpha$  is in the cross terms. But as  $\Psi_1$  and  $\Psi_2$  do not overlap, neither do  $\Psi_1$  and  $\partial\Psi_2/\partial x$  nor  $\Psi_2$  and  $\partial\Psi_1/\partial x$ . This means that the cross

terms are zero. Hence  $\bar{p}$  doesn't depend on  $\alpha$  either. Finally, in a similar way one can see that none of the moments of position and momentum,  $\bar{x}^n \bar{p}^k$ , depend on  $\alpha$ .

We seem to have a paradox<sup>3-5</sup>: every observable related to the particle can be written as a function of the operators  $x$  and  $p$ , but since all the moments of  $x$  and  $p$  are independent of  $\alpha$ , all observables seem to be independent of  $\alpha$ . But this surely cannot be the case: at a later time, when the two wavepackets overlap, the interference pattern does depend on the relative phase, so this information must be encoded somewhere. But where?

The answer is: in the shift operator,  $e^{ipL/\hbar}$ . As its name implies, this operator shifts a particle by a finite distance  $L$ , which in our case is the distance between the two wavepackets. Taking, for simplicity, the two wavepackets to be identical up to a shift,  $\Psi_2(x) = \Psi_1(x + L)$ , the average of the shift operator is  $(1/2)e^{-i\alpha}$ , featuring a dependence on  $\alpha$ .

We thus arrive at a first significant conclusion: the information about the interference is contained in the average of the shift operator, not in the averages of any power of position or momentum. It is therefore this operator that is the relevant one. Without a better understanding of its properties we have no way of actually understanding quantum interference.

Before proceeding, let us mention a technical point. The shift operator is non-Hermitian, and therefore cannot be an observable. The actual observables are its Hermitian components,  $\sin(pL/\hbar)$  and  $\cos(pL/\hbar)$ , or, an even better choice, the 'modular momentum',  $p \bmod 2\pi\hbar/L$ . But for mathematical simplicity we will consider the shift operator itself.

Let us now analyse the properties of the shift operator. First, it is obviously non-local: it shifts the particle by a finite distance  $L$ . Second, it is a purely quantum operator: it contains  $\hbar$  in such a manner that it becomes ill-defined in the classical limit of  $\hbar \rightarrow 0$ . Hence we see that the way in which quantum interference disappears in the classical limit is by means of the disappearance of its relevant observables.

Finally, let us study the time evolution of the shift operator. To put things into perspective, consider first the evolution of the position and momentum operators,  $x$  and  $p$ . The corresponding Heisenberg equations of motion,  $dx/dt = p/m$  and  $dp/dt = -dV(x)/dx$ , where  $m$  is the mass of the particle and  $V(x)$  the potential acting on it, look identical to their classical counterparts. The only difference is that quantum mechanically,  $x$  and  $p$  are

operators, whereas classically they are real numbers. This fact may create the impression that quantum and classical dynamics are actually very similar, and that the only difference is in the nature of the quantum state, that is, in the fact that quantum mechanically we cannot prepare a state with well-defined position and momentum. Nothing could be further from the truth.

This becomes clear when looking at the evolution of the shift operator. Classically, the time evolution of  $e^{ipL/\hbar}$  is

$$\frac{d e^{ipL/\hbar}}{dt} = \frac{d e^{ipL/\hbar}}{dp} \frac{dp}{dt} = -i \frac{L}{\hbar} e^{ipL/\hbar} \frac{dV(x)}{dx}$$

Quantum mechanically, however, as we are dealing with operators and not ordinary functions, we are not allowed to use the chain rule for differentiation as above. Instead, we have to solve directly the corresponding Heisenberg equation, which leads to

$$\frac{d e^{ipL/\hbar}}{dt} = -i \frac{L}{\hbar} e^{ipL/\hbar} \frac{V(x+L) - V(x)}{L}$$

For the shift operator the classical and quantum equations of motion are completely different. The classical one is local whereas the quantum one is non-local — it depends on the potential difference between two remote places,  $V(x+L)$  and  $V(x)$ . This is nothing other than the scalar AB effect, but here in the unexpected context of the two-slit experiment.

### New dynamics ahead?

To summarize, quantum interference is not the benign phenomenon that it is generally considered to be. It is

fundamentally different from classical interference. The relevant observables are non-local and they obey non-local equations of motion.

But the significance of this new way of looking at quantum evolution really comes into true force when we go beyond single-particle evolution and consider interactions between particles. In the light of what was just said, it should be unsurprising that there is a non-local aspect to every interaction. We just have to follow the relevant observables. Particles can interact at a distance and exchange momentum in the process, but what is involved is only the modular part of the momentum. This is, incidentally, how an electron exchanges momentum with the solenoid in the standard AB effect.

The implications are dramatic. On the one hand, the analysis described here sheds new light on the mystery of interference, to the extent that it should change the whole discourse. Rather than focusing on what happens at the screen — wavefunction collapse and so on — one should go back and revisit the notion that an electron, while passing through one slit, cannot know what happens to the slit not taken. It certainly can know, if the physics is non-local. But the implications do not stop here, at the subject of 'interpretations of quantum mechanics'. It is much more than that.

The entire quantum evolution is subject to dynamical non-locality, so we should be able to see its consequences in various quantum effects, regardless of their interpretation. A few such effects have already been presented<sup>3,4</sup>, but this is certainly just a beginning. In a sense, the situation is reminiscent of the status of Bell inequalities before the advent

of quantum information. For whoever seriously considered them, it was clear that they represented a fundamental aspect of quantum mechanics, and that there was no hope of obtaining a deep understanding of quantum physics while ignoring them. However a 'killer application' was missing; this only came thirty years later in the form of quantum information.

Conceptually however we are almost there. That non-local interactions exist and do not conflict with causality is only possible under the umbrella of quantum uncertainties. This, if we turn the table around, may give a whole new meaning to the reasons that uncertainties exist in the first place — so that nature may be non-local.

Quantum mechanics is by now more than eight decades old, but it is fair to say that we are still lacking a deep and intuitive understanding of its nature. The fact that new and puzzling quantum effects are frequently discovered is ample evidence for this: they are only puzzling because we do not have the appropriate conceptual tools to predict them. We have yet to develop them. In my opinion, appreciating that quantum evolution is non-local is one *sine qua non* ingredient.  $\square$

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